Mathematica 11.3 Integration Test Results

Test results for the 227 problems in "5.2.2 (d x)^m (a+b arccos(c x))^n.m"

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCos}[a\,x]^3}{x^4}\,dx$$

Optimal (type 4, 192 leaves, 14 steps):

$$-\frac{a^2\,\text{ArcCos}[a\,x]}{x}+\frac{a\,\sqrt{1-a^2\,x^2}\,\text{ArcCos}[a\,x]^2}{2\,x^2}-\frac{\text{ArcCos}[a\,x]^3}{3\,x^3}-$$
$$i\,a^3\,\text{ArcCos}[a\,x]^2\,\text{ArcTan}\left[e^{i\,\text{ArcCos}[a\,x]}\right]+a^3\,\text{ArcTanh}\left[\sqrt{1-a^2\,x^2}\right]+$$
$$i\,a^3\,\text{ArcCos}[a\,x]\,\text{PolyLog}\left[2,-i\,e^{i\,\text{ArcCos}[a\,x]}\right]-i\,a^3\,\text{ArcCos}[a\,x]\,\text{PolyLog}\left[2,i\,e^{i\,\text{ArcCos}[a\,x]}\right]-$$
$$a^3\,\text{PolyLog}\left[3,-i\,e^{i\,\text{ArcCos}[a\,x]}\right]+a^3\,\text{PolyLog}\left[3,i\,e^{i\,\text{ArcCos}[a\,x]}\right]$$

Result (type 4, 509 leaves):

$$\frac{1}{2} a^3 \Big(\text{ArcCos}[a x]^2 \text{Log}\left[1 - i e^{i \text{ArcCos}[a x]}\right] - \text{ArcCos}[a x]^2 \text{Log}\left[1 + i e^{i \text{ArcCos}[a x]}\right] +$$
$$\pi \text{ArcCos}[a x] \text{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) e^{-\frac{1}{2} i \text{ArcCos}[a x]} \left(-i + e^{i \text{ArcCos}[a x]}\right)\right] -$$
$$\text{ArcCos}[a x]^2 \text{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) e^{-\frac{1}{2} i \text{ArcCos}[a x]} \left(-i + e^{i \text{ArcCos}[a x]}\right)\right] +$$
$$\pi \text{ArcCos}[a x] \text{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i \text{ArcCos}[a x]} \left((1 + i) + (1 - i) e^{i \text{ArcCos}[a x]}\right)\right] +$$
$$\text{ArcCos}[a x]^2 \text{Log}\left[\frac{1}{2} e^{-\frac{1}{2} i \text{ArcCos}[a x]} \left((1 + i) + (1 - i) e^{i \text{ArcCos}[a x]}\right)\right] -$$
$$2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] - \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] +$$
$$\text{ArcCos}[a x]^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] - \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] -$$
$$\pi \text{ArcCos}[a x] \text{Log}\left[-\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] + \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] +$$
$$2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] + \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] -$$
$$\pi \text{ArcCos}[a x] \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] + \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] -$$
$$\text{ArcCos}[a x]^2 \text{Log}\left[\text{Cos}\left[\frac{1}{2} \text{ArcCos}[a x]\right] + \text{Sin}\left[\frac{1}{2} \text{ArcCos}[a x]\right]\right] +$$
$$2 i \text{ArcCos}[a x] \text{PolyLog}\left[2, -i e^{i \text{ArcCos}[a x]}\right] - 2 i \text{ArcCos}[a x] \text{PolyLog}\left[2, i e^{i \text{ArcCos}[a x]}\right] -$$
$$2 \text{PolyLog}\left[3, -i e^{i \text{ArcCos}[a x]}\right] + 2 \text{PolyLog}\left[3, i e^{i \text{ArcCos}[a x]}\right]\Big) -$$
$$\frac{\text{ArcCos}[a x] \left(12 a^2 x^2 + 4 \text{ArcCos}[a x]^2 - 3 \text{ArcCos}[a x] \text{Sin}[2 \text{ArcCos}[a x]]\right)}{12 x^3}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCos}[a x]^4}{x^2} \, dx$$

Optimal (type 4, 176 leaves, 11 steps):

$$-\frac{\text{ArcCos}[a x]^4}{x} - 8 i a \text{ArcCos}[a x]^3 \text{ArcTan}\left[e^{i \text{ArcCos}[a x]}\right] +$$
$$12 i a \text{ArcCos}[a x]^2 \text{PolyLog}\left[2, -i e^{i \text{ArcCos}[a x]}\right] - 12 i a \text{ArcCos}[a x]^2 \text{PolyLog}\left[2, i e^{i \text{ArcCos}[a x]}\right] -$$
$$24 a \text{ArcCos}[a x] \text{PolyLog}\left[3, -i e^{i \text{ArcCos}[a x]}\right] + 24 a \text{ArcCos}[a x] \text{PolyLog}\left[3, i e^{i \text{ArcCos}[a x]}\right] -$$
$$24 i a \text{PolyLog}\left[4, -i e^{i \text{ArcCos}[a x]}\right] + 24 i a \text{PolyLog}\left[4, i e^{i \text{ArcCos}[a x]}\right]$$

Result (type 4, 549 leaves):

$$a\left(-\frac{7 i \pi^4}{16}-\frac{1}{2} i \pi^3 \text{ArcCos}[a x]+\frac{3}{2} i \pi^2 \text{ArcCos}[a x]^2-2 i \pi \text{ArcCos}[a x]^3+i \text{ArcCos}[a x]^4-\frac{\text{ArcCos}[a x]^4}{a x}+3 \pi^2 \text{ArcCos}[a x] \text{Log}\left[1-i e^{-i \text{ArcCos}[a x]}\right]-6 \pi \text{ArcCos}[a x]^2 \text{Log}\left[1-i e^{-i \text{ArcCos}[a x]}\right]-\frac{1}{2} \pi^3 \text{Log}\left[1+i e^{-i \text{ArcCos}[a x]}\right]+4 \text{ArcCos}[a x]^3 \text{Log}\left[1+i e^{-i \text{ArcCos}[a x]}\right]+\frac{1}{2} \pi^3 \text{Log}\left[1+i e^{i \text{ArcCos}[a x]}\right]-3 \pi^2 \text{ArcCos}[a x] \text{Log}\left[1+i e^{i \text{ArcCos}[a x]}\right]+6 \pi \text{ArcCos}[a x]^2 \text{Log}\left[1+i e^{i \text{ArcCos}[a x]}\right]-4 \text{ArcCos}[a x]^3 \text{Log}\left[1+i e^{i \text{ArcCos}[a x]}\right]+\frac{1}{2} \pi^3 \text{Log}\left[\text{Tan}\left[\frac{1}{4} (\pi+2 \text{ArcCos}[a x])\right]\right]+12 i \text{ArcCos}[a x]^2 \text{PolyLog}\left[2,-i e^{-i \text{ArcCos}[a x]}\right]+3 i \pi (\pi-4 \text{ArcCos}[a x]) \text{PolyLog}\left[2, i e^{-i \text{ArcCos}[a x]}\right]+3 i \pi^2 \text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]-12 i \pi \text{ArcCos}[a x] \text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]+12 i \text{ArcCos}[a x]^2 \text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]+24 \text{ArcCos}[a x] \text{PolyLog}\left[3,-i e^{-i \text{ArcCos}[a x]}\right]-12 \pi \text{PolyLog}\left[3, i e^{-i \text{ArcCos}[a x]}\right]+12 \pi \text{PolyLog}\left[3,-i e^{i \text{ArcCos}[a x]}\right]-24 \text{ArcCos}[a x] \text{PolyLog}\left[3,-i e^{i \text{ArcCos}[a x]}\right]-24 i \text{PolyLog}\left[4,-i e^{-i \text{ArcCos}[a x]}\right]-24 i \text{PolyLog}\left[4,-i e^{i \text{ArcCos}[a x]}\right]\right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCos}[a x]^4}{x^4} \, dx$$

Optimal (type 4, 304 leaves, 19 steps):

$$-\frac{2 a^2 \text{ArcCos}[a x]^2}{x}+\frac{2 a \sqrt{1-a^2 x^2} \text{ArcCos}[a x]^3}{3 x^2}-\frac{\text{ArcCos}[a x]^4}{3 x^3}-8 i a^3 \text{ArcCos}[a x] \text{ArcTan}\left[e^{i \text{ArcCos}[a x]}\right]-\frac{4}{3} i a^3 \text{ArcCos}[a x]^3 \text{ArcTan}\left[e^{i \text{ArcCos}[a x]}\right]+4 i a^3 \text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]+2 i a^3 \text{ArcCos}[a x]^2 \text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]-4 i a^3 \text{PolyLog}\left[2, i e^{i \text{ArcCos}[a x]}\right]-2 i a^3 \text{ArcCos}[a x]^2 \text{PolyLog}\left[2, i e^{i \text{ArcCos}[a x]}\right]-4 a^3 \text{ArcCos}[a x] \text{PolyLog}\left[3,-i e^{i \text{ArcCos}[a x]}\right]+4 a^3 \text{ArcCos}[a x] \text{PolyLog}\left[3, i e^{i \text{ArcCos}[a x]}\right]-4 i a^3 \text{PolyLog}\left[4,-i e^{i \text{ArcCos}[a x]}\right]+4 i a^3 \text{PolyLog}\left[4, i e^{i \text{ArcCos}[a x]}\right]$$

Result (type 4, 1475 leaves):

$$a^3\left(-\frac{1}{6} \text{ArcCos}[a x]^2 \left(12+\text{ArcCos}[a x]^2\right)+4 \left(\text{ArcCos}[a x] \left(\text{Log}\left[1-i e^{i \text{ArcCos}[a x]}\right]-\text{Log}\left[1+i e^{i \text{ArcCos}[a x]}\right]\right)+i \left(\text{PolyLog}\left[2,-i e^{i \text{ArcCos}[a x]}\right]-\text{PolyLog}\left[2, i e^{i \text{ArcCos}[a x]}\right]\right)\right)+\frac{2}{3} \left(\frac{1}{8} \pi^3 \text{Log}\left[\text{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2}-\text{ArcCos}[a x]\right)\right]\right]+\frac{3}{4} \pi^2 \left(\left(\frac{\pi}{2}-\text{ArcCos}[a x]\right) \left(\text{Log}\left[1-e^{i \left(\frac{\pi}{2}-\text{ArcCos}[a x]\right)}\right]-\text{Log}\left[1+e^{i \left(\frac{\pi}{2}-\text{ArcCos}[a x]\right)}\right]\right)+i \left(\text{PolyLog}\left[2,-e^{i \left(\frac{\pi}{2}-\text{ArcCos}[a x]\right)}\right]-\text{PolyLog}\left[2, e^{i \left(\frac{\pi}{2}-\text{ArcCos}[a x]\right)}\right]\right)\right)-\right.\right.$$

$$\frac{3}{2}\pi\left(\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)^2\left(\text{Log}\left[1-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]-\text{Log}\left[1+e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]\right)+\right.$$
$$2\,i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)\left(\text{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]-\text{PolyLog}\left[2,e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]\right)+$$
$$\left.2\left(-\text{PolyLog}\left[3,-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]+\text{PolyLog}\left[3,e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]\right)\right)+$$
$$8\left(\frac{1}{64}i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)^4+\frac{1}{4}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^4-\right.$$
$$\frac{1}{8}\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)^3\text{Log}\left[1+e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]-$$
$$\frac{1}{8}\pi^3\left(i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)-\text{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]\right)-$$
$$\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^3\text{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]+$$
$$\frac{3}{8}i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)^2\text{PolyLog}\left[2,-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]+$$
$$\frac{3}{4}\pi^2\left(\frac{1}{2}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^2-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)\right.$$
$$\left.\text{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]+\frac{1}{2}i\,\text{PolyLog}\left[2,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]\right)+$$
$$\frac{3}{2}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^2\text{PolyLog}\left[2,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]-$$
$$\frac{3}{4}\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)\text{PolyLog}\left[3,-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]-$$
$$\frac{3}{2}\pi\left(\frac{1}{3}i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^3-\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)^2\right.$$
$$\text{Log}\left[1+e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]+i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)$$
$$\left.\text{PolyLog}\left[2,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]-\frac{1}{2}\text{PolyLog}\left[3,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]\right)-$$
$$\frac{3}{2}\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)\text{PolyLog}\left[3,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]-$$
$$\left.\frac{3}{4}i\,\text{PolyLog}\left[4,-e^{i\left(\frac{\pi}{2}-\text{ArcCos}[a\,x]\right)}\right]-\frac{3}{4}i\,\text{PolyLog}\left[4,-e^{2i\left(\frac{\pi}{2}+\frac{1}{2}\left(-\frac{\pi}{2}+\text{ArcCos}[a\,x]\right)\right)}\right]\right)\right)-$$
$$\frac{-4\,\text{ArcCos}[a\,x]^3+\text{ArcCos}[a\,x]^4}{12\left(\text{Cos}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]-\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]\right)^2}-$$
$$\frac{\text{ArcCos}[a\,x]^4\,\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]}{6\left(\text{Cos}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]-\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]\right)^3}+$$
$$\frac{\text{ArcCos}[a\,x]^4\,\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]}{6\left(\text{Cos}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]+\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]\right)^3}-$$
$$\frac{4\,\text{ArcCos}[a\,x]^3+\text{ArcCos}[a\,x]^4}{12\left(\text{Cos}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]+\text{Sin}\left[\frac{1}{2}\text{ArcCos}[a\,x]\right]\right)^2}-$$

$$\left(-12\,\text{ArcCos}[a\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]-\text{ArcCos}[a\,x]^4\,\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]\right)\Big/$$
$$\left(6\,\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]+\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]\right)\right)-$$
$$\left(12\,\text{ArcCos}[a\,x]^2\,\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]+\text{ArcCos}[a\,x]^4\,\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]\right)\Big/$$
$$\left.\left(6\,\left(\text{Cos}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]-\text{Sin}\left[\frac{1}{2}\,\text{ArcCos}[a\,x]\right]\right)\right)\right)$$

Problem 121: Unable to integrate problem.

$$\int (b\,x)^m\,\text{ArcCos}[a\,x]^2\,dx$$

Optimal (type 5, 150 leaves, 2 steps):

$$\frac{(b\,x)^{1+m}\,\text{ArcCos}[a\,x]^2}{b\,(1+m)}+\frac{2\,a\,(b\,x)^{2+m}\,\text{ArcCos}[a\,x]\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2\,x^2\right]}{b^2\,(1+m)\,(2+m)}+$$
$$\left(2\,a^2\,(b\,x)^{3+m}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2}\right\},\left\{2+\frac{m}{2},\frac{5}{2}+\frac{m}{2}\right\},a^2\,x^2\right]\right)\Big/$$
$$\left(b^3\,(1+m)\,(2+m)\,(3+m)\right)$$

Result (type 8, 14 leaves):

$$\int (b\,x)^m\,\text{ArcCos}[a\,x]^2\,dx$$

Problem 157: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b\,\text{ArcCos}[c\,x])^3}{x^2}\,dx$$

Optimal (type 4, 151 leaves, 9 steps):

$$-\frac{(a+b\,\text{ArcCos}[c\,x])^3}{x}-6\,i\,b\,c\,(a+b\,\text{ArcCos}[c\,x])^2\,\text{ArcTan}\left[e^{i\,\text{ArcCos}[c\,x]}\right]+$$
$$6\,i\,b^2\,c\,(a+b\,\text{ArcCos}[c\,x])\,\text{PolyLog}\left[2,-i\,e^{i\,\text{ArcCos}[c\,x]}\right]-$$
$$6\,i\,b^2\,c\,(a+b\,\text{ArcCos}[c\,x])\,\text{PolyLog}\left[2,i\,e^{i\,\text{ArcCos}[c\,x]}\right]-$$
$$6\,b^3\,c\,\text{PolyLog}\left[3,-i\,e^{i\,\text{ArcCos}[c\,x]}\right]+6\,b^3\,c\,\text{PolyLog}\left[3,i\,e^{i\,\text{ArcCos}[c\,x]}\right]$$

Result (type 4, 308 leaves):

$$-\frac{a^3}{x}-\frac{3 a^2 b \operatorname{ArcCos}[c x]}{x}-3 a^2 b c \operatorname{Log}[x]+3 a^2 b c \operatorname{Log}\left[1+\sqrt{1-c^2 x^2}\right]+$$
$$3 a b^2 c\left(-\frac{\operatorname{ArcCos}[c x]^2}{c x}+2\left(\operatorname{ArcCos}[c x]\left(\operatorname{Log}\left[1-i e^{i \operatorname{ArcCos}[c x]}\right]-\operatorname{Log}\left[1+i e^{i \operatorname{ArcCos}[c x]}\right]\right)+\right.\right.$$
$$\left.\left.i\left(\operatorname{PolyLog}\left[2,-i e^{i \operatorname{ArcCos}[c x]}\right]-\operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcCos}[c x]}\right]\right)\right)\right)+$$
$$b^3 c\left(-\frac{\operatorname{ArcCos}[c x]^3}{c x}+3\left(\operatorname{ArcCos}[c x]^2\left(\operatorname{Log}\left[1-i e^{i \operatorname{ArcCos}[c x]}\right]-\operatorname{Log}\left[1+i e^{i \operatorname{ArcCos}[c x]}\right]\right)+\right.\right.$$
$$2 i \operatorname{ArcCos}[c x]\left(\operatorname{PolyLog}\left[2,-i e^{i \operatorname{ArcCos}[c x]}\right]-\operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcCos}[c x]}\right]\right)-$$
$$\left.\left.2\left(\operatorname{PolyLog}\left[3,-i e^{i \operatorname{ArcCos}[c x]}\right]-\operatorname{PolyLog}\left[3, i e^{i \operatorname{ArcCos}[c x]}\right]\right)\right)\right)$$

Problem 203: Result unnecessarily involves imaginary or complex numbers.

$$\int (d x)^{5/2} (a+b \operatorname{ArcCos}[c x]) \, dx$$

Optimal (type 4, 120 leaves, 5 steps):

$$-\frac{20 b d^2 \sqrt{d x} \sqrt{1-c^2 x^2}}{147 c^3}-\frac{4 b (d x)^{5/2} \sqrt{1-c^2 x^2}}{49 c}+$$
$$\frac{2 (d x)^{7/2} (a+b \operatorname{ArcCos}[c x])}{7 d}+\frac{20 b d^{5/2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{147 c^{7/2}}$$

Result (type 4, 158 leaves):

$$\frac{1}{147 c^3 \sqrt{1-c^2 x^2}}$$
$$2 d^2 \sqrt{d x}\left(-10 b+4 b c^2 x^2+6 b c^4 x^4+21 a c^3 x^3 \sqrt{1-c^2 x^2}+21 b c^3 x^3 \sqrt{1-c^2 x^2} \operatorname{ArcCos}[c x]+\right.$$
$$\left.\frac{10 i b \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{x} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right],-1\right]}{\sqrt{-\frac{1}{c}}}\right)$$

Problem 204: Result unnecessarily involves imaginary or complex numbers.

$$\int (d x)^{3/2} (a+b \operatorname{ArcCos}[c x]) \, dx$$

Optimal (type 4, 124 leaves, 7 steps):

$$-\frac{4 b (d x)^{3/2} \sqrt{1-c^2 x^2}}{25 c}+\frac{2 (d x)^{5/2} (a+b \text{ArcCos}[c x])}{5 d}+\frac{12 b d^{3/2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{25 c^{5/2}}-\frac{12 b d^{3/2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{25 c^{5/2}}$$

Result (type 4, 107 leaves):

$$\frac{1}{25 c^2 \sqrt{-c x}} 2 d \sqrt{d x} \left(c x \sqrt{-c x} \left(5 a c x-2 b \sqrt{1-c^2 x^2}+5 b c x \text{ArcCos}[c x]\right)-6 i b \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-c x}\right],-1\right]+6 i b \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-c x}\right],-1\right]\right)$$

Problem 205: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{d x} (a+b \text{ArcCos}[c x]) \, dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$-\frac{4 b \sqrt{d x} \sqrt{1-c^2 x^2}}{9 c}+\frac{2 (d x)^{3/2} (a+b \text{ArcCos}[c x])}{3 d}+\frac{4 b \sqrt{d} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{9 c^{3/2}}$$

Result (type 4, 113 leaves):

$$\frac{2}{9} \sqrt{d x} \left(3 a x-\frac{2 b \sqrt{1-c^2 x^2}}{c}+3 b x \text{ArcCos}[c x]-\frac{2 i b \sqrt{-\frac{1}{c}} \sqrt{1-\frac{1}{c^2 x^2}} \sqrt{x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right],-1\right]}{\sqrt{1-c^2 x^2}}\right)$$

Problem 206: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \text{ArcCos}[c x]}{\sqrt{d x}} \, dx$$

Optimal (type 4, 89 leaves, 6 steps):

$$\frac{2 \sqrt{d x} (a+b \text{ArcCos}[c x])}{d}+\frac{4 b \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{\sqrt{c} \sqrt{d}}-\frac{4 b \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c} \sqrt{d x}}{\sqrt{d}}\right],-1\right]}{\sqrt{c} \sqrt{d}}$$

Result (type 4, 76 leaves):

$$\frac{1}{\sqrt{-c x}\ \sqrt{d x}} 2 x \left(\sqrt{-c x}\ (a + b \operatorname{ArcCos}[c x]) - 2 i b \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-c x}\right], -1\right] + 2 i b \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-c x}\right], -1\right]\right)$$

Problem 207: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCos}[c x]}{(d x)^{3/2}}\, dx$$

Optimal (type 4, 55 leaves, 3 steps):

$$-\frac{2 (a + b \operatorname{ArcCos}[c x])}{d \sqrt{d x}} - \frac{4 b \sqrt{c}\ \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c}\ \sqrt{d x}}{\sqrt{d}}\right], -1\right]}{d^{3/2}}$$

Result (type 4, 93 leaves):

$$\frac{1}{(d x)^{3/2}} 2 x \left(-a - b \operatorname{ArcCos}[c x] + \frac{2 i b \sqrt{-\frac{1}{c}}\ c^2 \sqrt{1 - \frac{1}{c^2 x^2}}\ x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right], -1\right]}{\sqrt{1 - c^2 x^2}}\right)$$

Problem 208: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcCos}[c x]}{(d x)^{5/2}}\, dx$$

Optimal (type 4, 125 leaves, 7 steps):

$$\frac{4 b c \sqrt{1 - c^2 x^2}}{3 d^2 \sqrt{d x}} - \frac{2 (a + b \operatorname{ArcCos}[c x])}{3 d (d x)^{3/2}} + \frac{4 b c^{3/2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c}\ \sqrt{d x}}{\sqrt{d}}\right], -1\right]}{3 d^{5/2}} - \frac{4 b c^{3/2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c}\ \sqrt{d x}}{\sqrt{d}}\right], -1\right]}{3 d^{5/2}}$$

Result (type 4, 110 leaves):

$$\frac{1}{3 \sqrt{-c x}\ (d x)^{5/2}} x \left(-2 \sqrt{-c x}\ \left(a - 2 b c x \sqrt{1 - c^2 x^2} + b \operatorname{ArcCos}[c x]\right) - 4 i b c^2 x^2 \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-c x}\right], -1\right] + 4 i b c^2 x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-c x}\right], -1\right]\right)$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int (d x)^{5/2} (a + b \operatorname{ArcCos}[c x])^2\, dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$\frac{2\,(d\,x)^{7/2}\,(a+b\,\text{ArcCos}[c\,x])^2}{7\,d}+\frac{1}{63\,d^2}$$
$$8\,b\,c\,(d\,x)^{9/2}\,(a+b\,\text{ArcCos}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2\,x^2\right]+$$
$$\frac{16\,b^2\,c^2\,(d\,x)^{11/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{11}{4},\frac{11}{4}\right\},\left\{\frac{13}{4},\frac{15}{4}\right\},c^2\,x^2\right]}{693\,d^3}$$

Result (type 5, 269 leaves):

$$\frac{1}{6174}$$
$$(d\,x)^{5/2}\left(1764\,a^2\,x+168\,a\,b\left(21\,x\,\text{ArcCos}[c\,x]+\left(2\,x\left(\sqrt{c\,x}\,\left(-5+2\,c^2\,x^2+3\,c^4\,x^4\right)-5\,c\,\sqrt{1-\frac{1}{c^2\,x^2}}\right.\right.\right.\right.$$
$$\left.\left.\left.x\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{c\,x}}\right],-1\right]\right)\right)\Big/\left((c\,x)^{7/2}\,\sqrt{1-c^2\,x^2}\right)\right)+$$
$$\left(b^2\left(4\,\text{Gamma}\left[\frac{5}{4}\right]\,\text{Gamma}\left[\frac{7}{4}\right]\left(-8\,c\,x\,\left(35+9\,c^2\,x^2\right)-84\,\sqrt{1-c^2\,x^2}\,\left(5+3\,c^2\,x^2\right)\,\text{ArcCos}[c\,x]+441\,c^3\right.\right.\right.$$
$$\left.x^3\,\text{ArcCos}[c\,x]^2+420\,\sqrt{1-c^2\,x^2}\,\text{ArcCos}[c\,x]\,\text{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{5}{4},c^2\,x^2\right]\right)+$$
$$\left.\left.210\,\sqrt{2}\,c\,\pi\,x\,\text{HypergeometricPFQ}\left[\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2\,x^2\right]\right)\right)\Big/\left(c^3\right.$$
$$\left.\left.x^2\,\text{Gamma}\left[\frac{5}{4}\right]\,\text{Gamma}\left[\frac{7}{4}\right]\right)\right)$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int\sqrt{d\,x}\,(a+b\,\text{ArcCos}[c\,x])^2\,dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$\frac{2\,(d\,x)^{3/2}\,(a+b\,\text{ArcCos}[c\,x])^2}{3\,d}+$$
$$\frac{8\,b\,c\,(d\,x)^{5/2}\,(a+b\,\text{ArcCos}[c\,x])\,\text{Hypergeometric2F1}\left[\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2\,x^2\right]}{15\,d^2}+$$
$$\frac{16\,b^2\,c^2\,(d\,x)^{7/2}\,\text{HypergeometricPFQ}\left[\left\{1,\frac{7}{4},\frac{7}{4}\right\},\left\{\frac{9}{4},\frac{11}{4}\right\},c^2\,x^2\right]}{105\,d^3}$$

Result (type 5, 228 leaves):

$$\frac{1}{27}\sqrt{d x}\left(18 a^2 x+36 a b x \operatorname{ArcCos}[c x]-\frac{24 b^2 \sqrt{1-c^2 x^2}\operatorname{ArcCos}[c x]}{c}+2 b^2 x\left(-8+9 \operatorname{ArcCos}[c x]^2\right)+\right.$$
$$\left(24 a b x\left(-\sqrt{c x}+(c x)^{5/2}-c\sqrt{1-\frac{1}{c^2 x^2}}\, x \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{c x}}\right],-1\right]\right)\right)\Big/$$
$$\left((c x)^{3/2}\sqrt{1-c^2 x^2}\right)+\frac{24 b^2 \sqrt{1-c^2 x^2}\operatorname{ArcCos}[c x]\operatorname{Hypergeometric2F1}\left[\frac{3}{4},1,\frac{5}{4},c^2 x^2\right]}{c}+$$
$$\left.\frac{3\sqrt{2}\, b^2 \pi x \operatorname{HypergeometricPFQ}\left[\left\{\frac{3}{4},\frac{3}{4},1\right\},\left\{\frac{5}{4},\frac{7}{4}\right\},c^2 x^2\right]}{\operatorname{Gamma}\left[\frac{5}{4}\right]\operatorname{Gamma}\left[\frac{7}{4}\right]}\right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{ArcCos}[c x])^2}{(d x)^{5/2}}\, dx$$

Optimal (type 5, 109 leaves, 2 steps):

$$-\frac{2\left(a+b \operatorname{ArcCos}[c x]\right)^2}{3 d (d x)^{3/2}}+\frac{8 b c \left(a+b \operatorname{ArcCos}[c x]\right)\operatorname{Hypergeometric2F1}\left[-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2 x^2\right]}{3 d^2 \sqrt{d x}}+$$
$$\frac{16 b^2 c^2 \sqrt{d x}\operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{4},\frac{1}{4},1\right\},\left\{\frac{3}{4},\frac{5}{4}\right\},c^2 x^2\right]}{3 d^3}$$

Result (type 5, 242 leaves):

$$\frac{1}{36 (d x)^{5/2}\operatorname{Gamma}\left[\frac{7}{4}\right]\operatorname{Gamma}\left[\frac{9}{4}\right]}$$
$$x\left(-8 \operatorname{Gamma}\left[\frac{7}{4}\right]\operatorname{Gamma}\left[\frac{9}{4}\right]\left(3 a^2-24 b^2 c^2 x^2-12 a b c x\sqrt{1-c^2 x^2}+6 a b \operatorname{ArcCos}[c x]-\right.\right.$$
$$12 b^2 c x\sqrt{1-c^2 x^2}\operatorname{ArcCos}[c x]+3 b^2 \operatorname{ArcCos}[c x]^2-12 a b (c x)^{3/2}$$
$$\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{c x}\right],-1\right]+12 a b (c x)^{3/2}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{c x}\right],-1\right]-$$
$$\left.4 b^2 c^3 x^3\sqrt{1-c^2 x^2}\operatorname{ArcCos}[c x]\operatorname{Hypergeometric2F1}\left[1,\frac{5}{4},\frac{7}{4},c^2 x^2\right]\right)+$$
$$\left.3\sqrt{2}\, b^2 c^4 \pi x^4 \operatorname{HypergeometricPFQ}\left[\left\{1,\frac{5}{4},\frac{5}{4}\right\},\left\{\frac{7}{4},\frac{9}{4}\right\},c^2 x^2\right]\right)$$

Problem 216: Attempted integration timed out after 120 seconds.

$$\int \sqrt{d x}\,(a+b \operatorname{ArcCos}[c x])^3\, dx$$

Optimal (type 8, 67 leaves, 1 step):

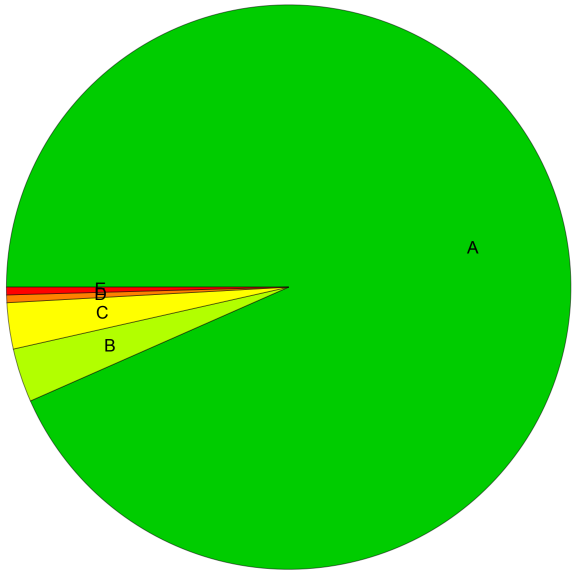
$$\frac{2\,(d\,x)^{3/2}\,(a+b\,\text{ArcCos}[c\,x])^3}{3\,d}+\frac{2\,b\,c\,\text{Int}\left[\frac{(d\,x)^{3/2}\,(a+b\,\text{ArcCos}[c\,x])^2}{\sqrt{1-c^2\,x^2}},\,x\right]}{d}$$

Result (type 1, 1 leaves):

???

Summary of Integration Test Results

227 integration problems



A - 212 optimal antiderivatives

B - 7 more than twice size of optimal antiderivatives

C - 6 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 1 integration timeouts